Mark Scheme (Results) June 2011

GCE Core Mathematics C2 (6664) Paper 1

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## EDEXCEL GCE MATHEMATICS

## General Instructions for Marking

1. The total number of marks for the paper is 75 .
2. The Edexcel Mathematics mark schemes use the following types of marks:

- M marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of $M$ marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod - benefit of doubt
- ft - follow through
- the symbol will be used for correct ft
- cao - correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw - ignore subsequent working
- awrt - answers which round to
- SC: special case
- oe - or equivalent (and appropriate)
- dep - dependent
- indep - independent
- dp decimal places
- sf significant figures
-     * The answer is printed on the paper
- $\quad$ The second mark is dependent on gaining the first mark

June 2011
Core Mathematics C2 6664
Mark Scheme

\begin{tabular}{|c|c|}
\hline Question Number \& Scheme \(\quad\) Marks \\
\hline \begin{tabular}{l}
1. \\
(a)
\end{tabular} \& \[
\begin{aligned}
\& \mathrm{f}(x)=2 x^{3}-7 x^{2}-5 x+4 \\
\& \text { Remainder }=\mathrm{f}(1)=2-7-5+4=-6 \\
\& \quad=-6
\end{aligned}
\] \\
\hline (b) \& \begin{tabular}{lr|r|}
\begin{tabular}{l}
\(\mathrm{f}(-1)=2(-1)^{3}-7(-1)^{2}-5(-1)+4\) \\
and so \((x+1)\) is a factor.
\end{tabular} \& \begin{tabular}{rl} 
Attempts \(\mathrm{f}(-1)\). \& M1 \\
\cline { 2 - 3 }
\end{tabular} \& \begin{tabular}{r} 
( -1\()=0\) with no sign or substitution \\
errors and for conclusion.
\end{tabular}
\end{tabular} \begin{tabular}{ll} 
A1 \& [2]
\end{tabular} \\
\hline (c) \& \begin{tabular}{rl|r|}
\(\mathrm{f}(x)\) \& \(=\{(x+1)\}\left(2 x^{2}-9 x+4\right)\) \& M 1 A1 \\
\& \(=(x+1)(2 x-1)(x-4)\) \& dM1 A1 \\
(Note: Ignore the ePEN notation of \((b)(\) should be \((c))\) for the final three marks in this part). \& [4] \\
\hline
\end{tabular} \\
\hline (a)
(b)

(c) \& | M1 for attempting either $\mathrm{f}(1)$ or $\mathrm{f}(-1)$. Can be implied. Only one slip permitted. |
| :--- |
| M1 can also be given for an attempt (at least two "subtracting" processes) at long division to give a remainder which is independent of $x$. A1 can be given also for -6 seen at the bottom of long division working. Award A0 for a candidate who finds -6 but then states that the remainder is 6 . |
| Award M1A1 for -6 without any working. |
| M1: attempting only $\mathrm{f}(-1)$. A1: must correctly show $\mathrm{f}(-1)=0$ and give a conclusion in part (b) only. |
| Note: Stating "hence factor" or "it is a factor" or a "tick" or "QED" is fine for the conclusion. |
| Note also that a conclusion can be implied from a preamble, eg: "If $\mathrm{f}(-1)=0,(x+1)$ is a factor...." |
| Note: Long division scores no marks in part (b). The factor theorem is required. |
| $1^{\text {st }}$ M1: Attempts long division or other method, to obtain ( $2 x^{2} \pm a x \pm b$ ), $a \neq 0$, even with a remainder. |
| Working need not be seen as this could be done "by inspection." ( $2 x^{2} \pm a x \pm b$ ) must be seen in part (c) only. Award $1^{\text {st }} \mathrm{M} 0$ if the quadratic factor is clearly found from dividing $\mathrm{f}(x)$ by $(x-1)$. Eg. Some candidates use their $\left(2 x^{2}-5 x-10\right)$ in part (c) found from applying a long division method in part (a). |
| $1^{\text {st }} \mathrm{A} 1$ : For seeing $\left(2 x^{2}-9 x+4\right)$. |
| $2^{\text {nd }} \mathrm{dM} 1$ : Factorises a 3 term quadratic. (see rule for factorising a quadratic). This is dependent on the previous method mark being awarded. This mark can also be awarded if the candidate applies the quadratic formula correctly. |
| $2^{\text {nd }} \mathrm{A} 1$ : is cao and needs all three factors on one line. Ignore following work (such as a solution to a quadratic equation.) |
| Note: Some candidates will go from $\{(x+1)\}\left(2 x^{2}-9 x+4\right)$ to $\{x=-1\}, x=\frac{1}{2}, 4$, and not list all three factors. Award these responses M1A1M1A0. |
| Alternative: $1^{\text {st }} \mathrm{M} 1$ : For finding either $\mathrm{f}(4)=0$ or $\mathrm{f}\left(\frac{1}{2}\right)=0$. |
| $1^{\text {st }}$ A1: A second correct factor of usually $(x-4)$ or $(2 x-1)$ found. Note that any one of the other correct factors found would imply the $1^{\text {st }}$ M1 mark. |
| $2^{\text {nd }} \mathrm{dM} 1$ : For using two known factors to find the third factor, usually $(2 x \pm 1)$. |
| $2^{\text {nd }} \mathrm{A} 1$ for correct answer of $(x+1)(2 x-1)(x-4)$. |
| Alternative: (for the first two marks) |
| $1^{\text {st }}$ M1: Expands $(x+1)\left(2 x^{2}+a x+b\right)$ \{giving $\left.2 x^{3}+(a+2) x^{2}+(b+a) x+b\right\}$ then compare coefficients to find values for $a$ and $b . \quad 1^{\text {st }} \mathrm{A} 1: a=-9, b=4$ |
| Not dealing with a factor of 2: $(x+1)\left(x-\frac{1}{2}\right)(x-4)$ or $(x+1)\left(x-\frac{1}{2}\right)(2 x-8)$ scores M1A1M1A0. |
| Answer only, with one sign error: eg. $(x+1)(2 x+1)(x-4)$ or $(x+1)(2 x-1)(x+4)$ scores |
| M1A1M1A0. (c) Award M1A1M1A1 for Listing all three correct factors with no working. | <br>

\hline
\end{tabular}


(a) The terms can be "listed" rather than added. Ignore any extra terms.
$1^{\text {st }} \mathrm{B} 1$ : A constant term of 243 seen. Just writing (3) ${ }^{5}$ is B0.
$2^{\text {nd }} \mathrm{B} 1$ : Term must be simplified to $405 b x$ for B1. The $x$ is required for this mark. Note $405+b x$ is B0.

M1: For either the $x$ term or the $x^{2}$ term. Requires correct binomial coefficient in any form with the correct power of $x$, but the other part of the coefficient (perhaps including powers of 3 and/or $b$ ) may be wrong or missing.
Allow binomial coefficients such as $\binom{5}{2},\left(\frac{5}{2}\right),\binom{5}{1},\left(\frac{5}{1}\right),{ }^{5} \mathrm{C}_{2},{ }^{5} \mathrm{C}_{1}$.
A1: For either $270 b^{2} x^{2}$ or $270(b x)^{2}$. (If $270 b x^{2}$ follows $270(b x)^{2}$, isw and allow A1.)
Alternative:
Note that a factor of $3^{5}$ can be taken out first: $3^{5}\left(1+\frac{b x}{3}\right)^{5}$, but the mark scheme still applies.
Ignore subsequent working (isw): Isw if necessary after correct working:
e.g. $243+405 b x+270 b^{2} x^{2}+\ldots$ leading to $9+15 b x+10 b^{2} x^{2}+\ldots$ scores B1B1M1A1 isw.

Also note that full marks could also be available in part (b), here.
Special Case: Candidate writing down the first three terms in descending powers of $x$ usually get $(b x)^{5}+{ }^{5} \mathrm{C}_{4}(3)^{1}(b x)^{4}+{ }^{5} \mathrm{C}_{3}(3)^{2}(b x)^{3}+\ldots=b^{5} x^{5}+15 b^{4} x^{4}+90 b^{3} x^{3}+\ldots$
So award SC: B0B0M1A0 for either $\left({ }^{5} \mathrm{C}_{4} \times \ldots \times x^{4}\right)$ or $\left({ }^{5} \mathrm{C}_{3} \times \ldots \times x^{3}\right)$
(b) M1 for equating 2 times their coefficient of $x$ to the coefficient of $x^{2}$ to get an equation in $b$, or equating their coefficient of $x$ to 2 times that of $x^{2}$, to get an equation in $b$.
Allow this M mark even if the equation is trivial, providing their coefficients from part (a) have been used, eg: $2(405 b)=270 b$, but beware $b=3$ from this, which is A0.
An equation in $b$ alone is required:
e.g. $2(405 b) x=270 b^{2} x^{2} \Rightarrow b=3$ or similar will be Special Case SC: M1A0 (as equation in coefficients only is not seen here).
e.g. $2(405 b) x=270 b^{2} x^{2} \Rightarrow 2(405 b)=270 b^{2} \Rightarrow b=3$ will get M1A1 (as coefficients rather than terms have now been considered).
Note: Answer of 3 from no working scores M1A0.
Note: The mistake $k\left(1+\frac{b x}{3}\right)^{5}, k \neq 243$ would give a maximum of 3 marks: B0B0M1A0, M1A1
Note: For $270 b x^{2}$ in part (a), followed by $2(405 b)=270 b^{2} \Rightarrow b=3$, in part (b), allow recovery M1A1.

| Question Number | Scheme $\quad$ Marks |
| :---: | :---: |
| 3. $\begin{aligned} \\ \\ \text { (a) }\end{aligned}$ | $\begin{aligned} & \text { (a) } 5^{x}=10 \text { and (b) } \log _{3}(x-2)=-1 \\ & x=\frac{\log 10}{\log 5} \text { or } x=\log _{5} 10 \\ & x\{=1.430676558 \ldots\}=1.43(3 \mathrm{sf}) \end{aligned}$ |
| (b) | $(x-2)=3^{-1}$ $(x-2)=3^{-1}$ or $\frac{1}{3}$ M1 oe  <br> $x\left\{=\frac{1}{3}+2\right\}=2 \frac{1}{3}$ $2 \frac{1}{3}$ or $\frac{7}{3}$ or 2.3 or awrt 2.33 A1  <br>   $[2]$  |
| (a) (b) | M1: for $x=\frac{\log 10}{\log 5}$ or $x=\log _{5} 10$. Also allow M1 for $x=\frac{1}{\log 5}$ <br> 1.43 with no working (or any working) scores M1A1 (even if left as $5^{1.43}$ ). <br> Other answers which round to 1.4 with no working score M1A0. <br> Trial \& Improvement Method: M1: For a method of trial and improvement by trialing <br> $\mathrm{f}($ value between 1.4 and 1.43) $=$ Value below 10 and <br> $\mathrm{f}($ value between 1.431 and 1.5$)=$ Value over 10 . <br> A1 for 1.43 cao. <br> Note: $x=\log _{10} 5$ by itself is M0; but $x=\log _{10} 5$ followed by $x=1.430676558 \ldots$ is M1. <br> M1: Is for correctly eliminating log out of the equation. <br> Eg 1: $\log _{3}(x-2)=\log _{3}\left(\frac{1}{3}\right) \Rightarrow x-2=\frac{1}{3}$ only gets M1 when the logs are correctly removed. <br> Eg 2: $\log _{3}(x-2)=-\log _{3}(3) \Rightarrow \log _{3}(x-2)+\log _{3}(3)=0 \Rightarrow \log _{3}(3(x-2))=0$ <br> $\Rightarrow 3(x-2)=3^{0}$ only gets M1 when the logs are correctly removed, <br> but $3(x-2)=0$ would score M0. <br> Note: $\log _{3}(x-2)=-1 \Rightarrow \log _{3}\left(\frac{x}{2}\right)=-1 \Rightarrow \frac{x}{2}=3^{-1}$ would score M0 for incorrect use of logs. <br> Alternative: changing base $\frac{\log _{10}(x-2)}{\log _{10} 3}=-1 \Rightarrow \log _{10}(x-2)=-\log _{10} 3 \Rightarrow \log _{10}(x-2)+\log _{10} 3=0$ <br> $\Rightarrow \log _{10} 3(x-2)=0 \Rightarrow 3(x-2)=10^{0}$. At this point M1 is scored. <br> A correct answer in (b) without any working scores M1A1. |



Note: Please mark parts (a) and (b) together. Answers only in (a) and/or (b) get full marks. Note in part (a) the marks are now M1A1 and not B1B1 as on ePEN.
M1: for $( \pm 2, \pm 1)$. Otherwise, M1 for an attempt to complete the square $(y \pm 1)^{2} \pm \beta, \beta \neq 0$. M1A1: Correct answer of $(-2,1)$ stated from any working gets M1A1.
(b) M1: to find the radius using 11 , " 1 " and " 4 ", ie. $r=\sqrt{11 \pm " 1 " \pm " 4 "}$. By applying this method candidates will usually achieve $\sqrt{16}, \sqrt{6}, \sqrt{8}$ or $\sqrt{14}$ and not $16,6,8$ or 14 .
Note: $(x+2)^{2}+(y-1)^{2}=-11-5=-16 \Rightarrow r=\sqrt{16}=4$ should be awarded M0A0.
Alternative: M1 in part (a): For comparing with $x^{2}+y^{2}+2 g x+2 f y+c=0$ to write down centre $(-g,-f)$ directly. Condone sign errors for this M mark. M1 in part (b): For using $r=\sqrt{g^{2}+f^{2}-c}$. Condone sign errors for this method mark.
$(x+2)^{2}+(y-1)^{2}=16 \Rightarrow r=8$ scores M0A0, but $r=\sqrt{16}=8$ scores M1A1 isw.
(c) $\quad 1^{\text {st }}$ M1: Putting $x=0$ in either $x^{2}+y^{2}+4 x-2 y-11=0$ or their circle equation usually given in part (a) or part (b). $\quad 1^{\text {st }} \mathrm{A} 1$ for a correct equation in $y$ in any form which can be implied by later working.
$2^{\text {nd }}$ M1: See rules for using the formula. Or completing the square on a 3TQ to give $y=a \pm \sqrt{b}$, where $\sqrt{b}$ is a surd, $b \neq$ their 11 and $b>0$. This mark should not be given for an attempt to factorise. $2^{\text {nd }}$ A1: Need exact pair in simplified surd form of $\{y=\} 1 \pm 2 \sqrt{3}$. This mark is also cso.
Do not need to see $(0,1+2 \sqrt{3})$ and $(0,1-2 \sqrt{3})$. Allow $2^{\text {nd }}$ A1 for bod $(1+2 \sqrt{3}, 0)$ and $(1-2 \sqrt{3}, 0)$. Any incorrect working in (c) gets penalised the final accuracy mark. So, beware: incorrect $(x-2)^{2}+(y-1)^{2}=16$ leading to $y^{2}-2 y-11=0$ and then $y=1 \pm 2 \sqrt{3}$ scores M1A1M1A0.
Special Case for setting $y=0$ : Award SC: M0A0M1A0 for an attempt at applying the formula

$$
x=\frac{-4 \pm \sqrt{(-4)^{2}-4(1)(-11)}}{2(1)}\left\{=\frac{-4 \pm \sqrt{60}}{2}=-2 \pm \sqrt{15}\right\}
$$

Award SC: M0A0M1A0 for completing the square to their equation in $x$ which will usually be $x^{2}+4 x-11=0$ to give $a \pm \sqrt{b}$, where $\sqrt{b}$ is a surd, $b \neq$ their 11 and $b>0$.
Special Case: For a candidate not using $\pm$ but achieving one of the correct answers then award SC: M1A1 M1A0 for one of either $y=1+2 \sqrt{3}$ or $y=1-2 \sqrt{3}$ or $y=1+\sqrt{12}$ or $y=1-\sqrt{12}$.


| Question <br> Number | Scheme $\quad$ Marks |
| :---: | :---: |
| 6. (a) | $\begin{aligned} & \left\{a r=192 \text { and } a r^{2}=144\right\} \\ & r=\frac{144}{192} \\ & r=\frac{3}{4} \text { or } 0.75 \end{aligned}$ <br> Attempt to eliminate $a$. (See notes.) |
| (b) | $\begin{aligned} & a(0.75)=192 \\ & a\left\{=\frac{192}{0.75}\right\}=256 \end{aligned}$ |
| (c) | $\mathrm{S}_{\infty}=\frac{256}{1-0.75}$ Applies $\frac{a}{1-r}$ correctly using both their $a$ and their $\|r\|<1$. M1 <br> So, $\left\{\mathrm{S}_{\infty}=\right\} 1024$ 1024 A1 cao |
| (d) |  |
| (a) (b) | M1: for eliminating $\boldsymbol{a}$ by eg. $192 r=144$ or by either dividing $a r^{2}=144$ by $a r=192$ or dividing $a r=192$ by $a r^{2}=144$, to achieve an equation in $r$ or $\frac{1}{r}$ Note that $r^{2}-r=\frac{144}{192}$ is M0. <br> Note also that any of $r=\frac{144}{192}$ or $r=\frac{192}{144}\left\{=\frac{4}{3}\right\}$ or $\frac{1}{r}=\frac{192}{144}$ or $\frac{1}{r}=\frac{144}{192}$ are fine for the award of M1. Note: A candidate just writing $r=\frac{144}{192}$ with no reference to $a$ can also get the method mark. Note: $a r^{2}=192$ and $a r^{3}=144$ leading to $r=\frac{3}{4}$ scores M1A1. This is because $r$ is the ratio between any two consecutive terms. These candidates, however, will usually be penalised in part (b). M1 for inserting their $r$ into either of the correct equations of either $a r=192$ or $\{a=\} \frac{192}{r}$ or $a r^{2}=144$ or $\{a=\} \frac{144}{r^{2}}$. No slips allowed here for M1. <br> M1: can also be awarded for writing down $144=a\left(\frac{192}{a}\right)^{2}$ <br> A1 for $a=256$ only. Note 256 from any working scores M1A1. <br> Note: Some candidates incorrectly confuse notation to give $r=\frac{4}{3}$ or 1.33 in part (a) (getting M1A0). In part (b), they recover to write $a=192 \times \frac{4}{3}$ for M1 and then 256 for A1. |


| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| (c) | M1: for applying $\frac{a}{1-r}$ correctly (no slips allowed!) using both their $a$ and their $r$, where $\|r\|<1$. <br> A1: for 1024, cao. |  |
|  |  |  |
|  | In parts (a) or (b) or (c), the correct answer with no working scores full marks. <br> $1^{\text {st }} \mathrm{M} 1$ : For applying S with their $a$ and either "the letter $r$ " or their $r$ and "uses" 1000 . |  |
| (d) | $2^{\text {nd }}$ M1: For isolating $+(r)^{n}$ and not $(a r)^{n}$, (eg. (192) $)^{n}$ ) as the subject of an equation or inequality. |  |
|  | $3^{\text {rd }} \mathrm{M} 1$ : For applying the power law to $\lambda^{k}=\mu$ to give $k \log \lambda=\log \mu$ oe. where $\lambda, \mu>0$. or $3^{\text {rd }} \mathrm{M} 1$ : For solving $\lambda^{k}=\mu$ to give $k=\log _{\lambda} \mu$, where $\lambda, \mu>0$. |  |
|  | A1: cso If a candidate uses inequalities, a fully correct method with inequalities is required here. So, an incorrect inequality statement at any stage in a candidate's working for this part loses this mark. |  |
|  | Note: Some candidates do not realise that the direction of the inequality is reversed in the final line of their solution. |  |
|  | Or A1: cso Note a candidate can achieve full marks here if they do not use inequalities. So, if a candidate uses equations rather than inequalities in their working then they need to final line of their working that $n=13.04$ (truncated) or $n=$ awrt $13.05 \Rightarrow n=14$ for A1. $n=14$ from no working gets SC: M0M0M1A1. | ate in the |
|  | A method of $\mathrm{T}_{n}>1000 \Rightarrow 256(0.75)^{n-1}>1000$ can score M0M0M1A0 for a correct application of the power law of logarithms. <br> Trial \& Improvement Method: |  |
|  | For $a=256$ and $r=0.75$, apply the following scheme: |  |
|  | $\mathrm{S}_{13}=\frac{256\left(1-(0.75)^{13}\right)}{1-0.75}=999.6725616 \ldots \quad \begin{array}{r}\text { Attempt to find either } S_{13} \text { or } S_{14} . \\ \text { EITHER (1) } \mathrm{S}_{13}=\text { awrt } 999.7 \text { or truncated }\end{array}$ | . |
|  | $999 \text { OR (2) } \mathrm{S}_{14}=\underset{\text { truncated } 1005 .}{\text { awrt } 1005.8 \text { or }}$ | . |
|  |  | M1 |
|  | 999 AND (2) $\mathrm{S}_{14}=$ awrt 1005.8 or <br> So, $n=14$. truncated 1005 AND $n=14$. | A1 |

\begin{tabular}{|c|c|c|}
\hline Question Number \& Scheme \& Marks \\
\hline \& \begin{tabular}{l}
Note: A similar scheme would apply for T\&I for candidates using their \(a\) and their \(r\). So,... \(1^{\text {st }} \mathrm{M} 1\) : For attempting to find one of the correct \(\mathrm{S}_{n}\) 's either side (but next to) 1000 . \\
\(2^{\text {nd }}\) M1: For one of these \(S_{n}\) 's correct for their \(a\) and their \(r\). (You may need to get your ca out!) \\
\(3^{\text {rd }} \mathrm{M} 1\) : For attempting to find both of the correct \(S_{n}\) 's either side (but next to) 1000 . \\
A1: Cannot be gained for wrong \(a\) and/or \(r\). \\
Trial \& Improvement Cumulative Approach: \\
A similar scheme to T\&I will be applied here: \\
\(1^{\text {st }} \mathrm{M} 1\) : For getting as far as the cumulative sum of 13 terms. \(2^{\text {nd }} \mathrm{M} 1:(1) \mathrm{S}_{13}=\) awrt 999.7 truncated 999. \(3^{\text {rd }} \mathrm{M} 1\) : For getting as far as the cumulative sum to 14 terms. Also at this st \(\mathrm{S}_{13}<1000\) and \(\mathrm{S}_{14}>1000\). A1: BOTH (1) \(\mathrm{S}_{13}=\) awrt 999.7 or truncated 999 AND (2) \(\mathrm{S}_{14}=\) awrt 1005.8 or truncated 1005 AND \(n=14\). \\
Trial \& Improvement Method: for \((0.75)^{n}<\frac{6}{256}=0.0234375\) \\
\(3^{\text {rd }} \mathrm{M} 1\) : For evidence of examining both \(n=13\) and \(n=14\). \\
Eg: \((0.75)^{13}\{=0.023757 \ldots\}\) and \((0.75)^{14}\{=0.0178179 \ldots\}\) \\
A1: \(n=14\) \\
Any misreads, \(S_{n}>10000\) etc, please escalate up to your Team Leader.
\end{tabular} \& ulators \\
\hline 7.

$\quad(a)$ \& | (a) $3 \sin \left(x+45^{\circ}\right)=2 ; 0 \leq x<360^{\circ}$ |
| :--- |
| (b) $2 \sin ^{2} x+2=7 \cos x ; 0 \leq x<2 \pi$ $\sin \left(x+45^{\circ}\right)=\frac{2}{3}$, so $\left(x+45^{\circ}\right)=41.8103 \ldots \quad(\alpha=41.8103 \ldots) \quad \sin ^{-1}\left(\frac{2}{3}\right)$ or awrt 41.8 or awrt $0.73^{\text {c }}$ |
| So, $x+45^{\circ}=\{138.1897 \ldots, 401.8103 \ldots\}$ $x+45^{\circ}=$ either " $180-$ their $\alpha$ " or $" 360^{\circ}+$ their $\alpha "$ ( $\alpha$ could be in radians). |
| and $x=\{93.1897 \ldots, 356.8103 \ldots\}$ |
| Either awrt $93.2^{\circ}$ or awrt $356.8^{\circ}$ |
| Both awrt $93.2^{\circ}$ and awrt $356.8^{\circ}$ | \& | M1 |  |
| :--- | :--- |
| M1 |  |
| A1 |  |
| A1 |  |
|  |  |
|  |  |
|  |  | <br>

\hline (b) \&  \& M1 A1 oe M1 A1 cso B1 B1 ft <br>
\hline
\end{tabular}

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| Question Number | Scheme $\quad$ Marks |
| :---: | :---: |
| (a) | $1^{\text {st }} \mathrm{M} 1$ : can also be implied for $x=$ awrt -3.2 <br> $2^{\text {nd }}$ M1: for $x+45^{\circ}=$ either " $180-$ their $\alpha$ " or " $360^{\circ}+$ their $\alpha$ ". This can be implied by later working. The candidate's $\alpha$ could also be in radians. <br> Note that this mark is not for $x=$ either " 180 - their $\alpha$ " or " $360^{\circ}+$ their $\alpha$ ". <br> Note: Imply the first two method marks or award M1M1A1 for either awrt $93.2^{\circ}$ or awrt $356.8^{\circ}$. <br> Note: Candidates who apply the following incorrect working of $3 \sin \left(x+45^{\circ}\right)=2$ <br> $\Rightarrow 3(\sin x+\sin 45)=2$, etc will usually score M0M0A0A0. <br> If there are any EXTRA solutions inside the range $0 \leq x<360$ and the candidate would otherwise score FULL MARKS then withhold the final aA2 mark (the final mark in this part of the question). Also ignore EXTRA solutions outside the range $0 \leq x<360$. <br> Working in Radians: Note the answers in radians are $x=$ awrt 1.6, awrt 6.2 <br> If a candidate works in radians then mark part (a) as above awarding the A marks in the same way. If the candidate would then score FULL MARKS then withhold the final aA2 mark (the final mark in this part of the question.) <br> No working: Award M1M1A1A0 for one of awrt $93.2^{\circ}$ or awrt $356.8^{\circ}$ seen without any working. Award M1M1A1A1 for both awrt $93.2^{\circ}$ and awrt $356.8^{\circ}$ seen without any working. <br> Allow benefit of the doubt (FULL MARKS) for final answer of $\sin x\{$ and not $x\}=\{$ awrt 93.2, awrt 356.8\} |

:

| Question <br> Number | Scheme | Marks |
| :---: | :--- | :--- |
| (b) | $1^{\text {st }}$ M1: for a correct method to use $\sin ^{2} x=1-\cos ^{2} x$ on the given equation. <br>  <br>  <br>  <br> Give bod if the candidate omits the bracket when substituting for $\sin ^{2} x$, but <br> $2-\cos ^{2} x+2=7 \cos x$, without supporting working, (eg. seeing " $\sin ^{2} x=1-\cos ^{2} x$ ") would score <br> $1^{\text {st }} \mathrm{M} 0$. |  |

Note that applying $\sin ^{2} x=\cos ^{2} x-1$, scores M0.
$1^{\text {st }} \mathrm{A} 1$ : for obtaining either $2 \cos ^{2} x+7 \cos x-4$ or $-2 \cos ^{2} x-7 \cos x+4$.
$1^{\text {st }} \mathrm{A} 1$ : can also awarded for a correct three term equation eg. $2 \cos ^{2} x+7 \cos x=4$ or
$2 \cos ^{2} x=4-7 \cos x$ etc.
$2^{\text {nd }} \mathrm{M} 1$ : for a valid attempt at factorisation of a quadratic (either 2 TQ or 3 TQ ) in cos, can use any variable here, $c, y, x$ or $\cos x$, and an attempt to find at least one of the solutions. See introduction to the Mark Scheme. Alternatively, using a correct formula for solving the quadratic. Either the formula must be stated correctly or the correct form must be implied by the substitution.
$2^{\text {nd }} \mathrm{A} 1$ : for $\cos x=\frac{1}{2}$, BY A CORRECT SOLUTION ONLY UP TO THIS POINT. Ignore extra answer of $\cos x=-4$, but penalise if candidate states an incorrect result e.g. $\cos x=4$. If they have used a substitution, a correct value of their $c$ or their $y$ or their $x$.
Note: $2^{\text {nd }} \mathrm{A} 1$ for $\cos x=\frac{1}{2}$ can be implied by later working.
$1^{\text {st }}$ B1: for either $\frac{\pi}{3}$ or awrt $1.05^{\text {c }}$
$2^{\text {nd }} \mathrm{B} 1$ : for either $\frac{5 \pi}{3}$ or awrt $5.24^{\mathrm{c}}$ or can be ft from $2 \pi$ - their $\beta$ or $360^{\circ}-$ their $\beta$ where
$\beta=\cos ^{-1}(k)$, such that $0<k<1$ or $-1<k<0$, but $k \neq 0, k \neq 1$ or $k \neq-1$.
If there are any EXTRA solutions inside the range $0 \leq x<2 \pi$ and the candidate would otherwise score FULL MARKS then withhold the final bB2 mark (the final mark in this part of the question). Also ignore EXTRA solutions outside the range $0 \leq x<2 \pi$.
Working in Degrees: Note the answers in degrees are $x=60,300$
If a candidate works in degrees then mark part (b) as above awarding the B marks in the same way. If the candidate would then score FULL MARKS then withhold the final bB2 mark (the final mark in this part of the question.)
Answers from no working:
$x=\frac{\pi}{3}$ and $x=\frac{5 \pi}{3}$ scores M0A0M0A0B1B1,
$x=60$ and $x=300$ scores M0A0M0A0B1B0,
$x=\frac{\pi}{3}$ ONLY or $x=60$ ONLY scores M0A0M0A0B1B0,
$x=\frac{5 \pi}{3}$ ONLY or $x=120$ ONLY scores M0A0M0A0B0B1.
No working: You cannot apply the ft in the B1ft if the answers are given with NO working.
Eg: $x=\frac{\pi}{5}$ and $x=\frac{9 \pi}{3}$ FROM NO WORKING scores M0A0M0A0B0B0.
For candidates using trial \& improvement, please forward these to your Team Leader.

| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 8. <br> (a) | $\begin{array}{lr} \{V=\} 2 x^{2} y=81 & 2 x^{2} y=81 \\ \{L=2(2 x+x+2 x+x)+4 y \Rightarrow L=12 x+4 y\} & \\ y=\frac{81}{2 x^{2}} \Rightarrow L=12 x+4\left(\frac{81}{2 x^{2}}\right) & \begin{array}{l} \text { Making } y \text { the subject of their } \\ \text { expression and substitute this } \\ \text { into the correct } L \text { formula. } \end{array} \\ \quad \text { So, } L=12 x+\frac{162}{x^{2}} \text { AG } & \text { Correct solution only. AG. } \end{array}$ | B1 oe <br> M1 <br> A1 cso <br> [3] |
| (b) | $\left.\begin{array}{lr} \frac{\mathrm{d} L}{\mathrm{~d} x}=12-\frac{324}{x^{3}}\left\{=12-324 x^{-3}\right\} & \text { Either } 12 x \rightarrow 12 \text { or } \frac{162}{x^{2}} \rightarrow \frac{ \pm \lambda}{x^{3}} \\ \text { Correct differentiation (need not be simplified). } \\ L^{\prime}=0 \text { and "their } x^{3}= \pm \text { value" } \\ \left\{\frac{\mathrm{d} L}{\mathrm{~d} x}=\right\} 12-\frac{324}{x^{3}}=0 \Rightarrow x^{3}=\frac{324}{12} ;=27 \Rightarrow x=3 & \text { or "their } x^{-3}= \pm \text { value" } \\ x=\sqrt[3]{27} \text { or } x=3 \end{array}\right\} \begin{array}{r} \text { Substitute candidate's value of } \\ x(\neq 0) \text { into a formula for } L . \end{array}$ | M1 <br> A1 aef M1; <br> A1 cso <br> ddM1 <br> A1 cao <br> [6] |
| (c) | $\{$ For $x=3\}, \frac{\mathrm{d}^{2} L}{\mathrm{~d} x^{2}}=\frac{972}{x^{4}}>0 \Rightarrow$ Minimum $\quad$Correct $\mathrm{ft} L^{\prime \prime}$ and considering sign. <br> $\frac{972}{x^{4}}$ and $>0$ and conclusion. | $\begin{array}{ll} \hline \text { M1 } \\ \text { A1 } \end{array}$ |

B1: For any correct form of $2 x^{2} y=81$. (may be unsimplified). Note that $2 x^{3}=81$ is B0. Otherwise, candidates can use any symbol or letter in place of $y$.
M1: Making $y$ the subject of their formula and substituting this into a correct expression for $L$.
A1: Correct solution only. Note that the answer is given.
(b)

Note you can mark parts (b) and (c) together.
$2^{\text {nd }}$ M1: Setting their $\frac{\mathrm{d} L}{\mathrm{~d} x}=0$ and "candidate's ft correct power of $x=$ a value". The power of $x$ must be consistent with their differentiation. If inequalities are used this mark cannot be gained until candidate states value of $x$ or $L$ from their $x$ without inequalities.
$L^{\prime}=0$ can be implied by $12=\frac{324}{x^{3}}$.
$2^{\text {nd }} \mathrm{A} 1: x^{3}=27 \Rightarrow x= \pm 3$ scores A0.
$2^{\text {nd }} \mathrm{A} 1$ : can be given for no value of $x$ given but followed through by correct working leading to $L=54$.
$3^{\text {rd }}$ M1: Note that this method mark is dependent upon the two previous method marks being awarded.
(c)

M 1 : for attempting correct ft second derivative and considering its sign.
A1: Correct second derivative of $\frac{972}{x^{4}}$ (need not be simplified) and a valid reason (e.g. $>0$ ), and conclusion. Need to conclude minimum (allow $x$ and not $L$ is a minimum) or indicate by a tick that it is a minimum. The actual value of the second derivative, if found, can be ignored, although substituting their $L$ and not $x$ into $L^{\prime \prime}$ is A0. Note: 2 marks can be scored from a wrong value of $x$, no value of $x$ found or from not substituting in the value of their $x$ into $L^{\prime \prime}$.
Gradient test or testing values either side of their $x$ scores M0A0 in part (c).
Throughout this question allow confused notation such as $\frac{\mathrm{d} y}{\mathrm{~d} x}$ for $\frac{\mathrm{d} L}{\mathrm{~d} x}$.

## edexcel

| Question Number | Scheme | Marks |
| :---: | :---: | :---: |
| 9. <br> (a) | $\begin{aligned} & \text { Curve: } y=-x^{2}+2 x+24 \text {, Line: } y=x+4 \\ & \{\text { Curve }=\text { Line }\} \Rightarrow-x^{2}+2 x+24=x+4 \\ & x^{2}-x-20\{=0\} \Rightarrow(x-5)(x+4)\{=0\} \Rightarrow x=\ldots . . \end{aligned}$ <br> So, $x=5,-4$ <br> So corresponding $y$-values are $y=9$ and $y=0$. <br> Eliminating $y$ correctly. <br> Attempt to solve a resulting quadratic to give $x=$ their values. Both $x=5$ and $x=-4$. <br> See notes below. | B1 <br> M1 <br> A1 <br> B1ft [4] |
| (b) | $\begin{aligned} & \left\{\int\left(-x^{2}+2 x+24\right) \mathrm{d} x\right\}=-\frac{x^{3}}{3}+\frac{2 x^{2}}{2}+24 x\left\{+c \quad \begin{array}{r} \mathrm{M} 1: x^{n} \rightarrow x^{n+1} \text { for any one term. } \\ 1^{\text {st A1 At least two out of three terms. }} \begin{array}{l} 2^{\text {nd }} \text { A1 for correct answer. } \end{array} \\ {\left[-\frac{x^{3}}{3}+\frac{2 x^{2}}{2}+24 x\right]_{-4}^{5}=(\ldots \ldots)-(\ldots \ldots) \quad \begin{array}{r} \text { Substitutes } 5 \text { and }-4 \text { (or their limits from } \\ \text { part(a)) into an "integrated function" and } \\ \text { subtracts, either way round. } \end{array}} \\ \left\{\left(-\frac{125}{3}+25+120\right)-\left(\frac{64}{3}+16-96\right)=\left(103 \frac{1}{3}\right)-\left(-58 \frac{2}{3}\right)=162\right\} \end{array}\right. \end{aligned}$ $\text { Area of } \Delta=\frac{1}{2}(9)(9)=40.5$ Uses correct method for finding area of triangle. $\text { So area of } R \text { is } 162-40.5=121.5$ Area under curve - Area of triangle. | M1A1A1 <br> dM1 <br> M1 <br> M1 <br> A1 oe cao |



| Question <br> Number | Scheme | Marks |
| :---: | :---: | :---: |
| Aliter <br> 9.(b) <br> Way 2 | Curve: $y=-x^{2}+2 x+24$, Line: $y=x+4$ <br> $3^{\text {rd }}$ M1: Uses integral of $(x+4)$ with <br> Area of $R=\int_{-4}^{5}\left(-x^{2}+2 x+24\right)-(x+4) d x$ $\begin{array}{lr} =-\frac{x^{3}}{3}+\frac{x^{2}}{2}+20 x\{+c\} & \text { M: } x^{n} \\ {\left[-\frac{x^{3}}{3}+\frac{x^{2}}{2}+20 x\right]_{-4}^{5}=(\ldots . .)-(\ldots . .)} & \text { A1 at } 1 \\ \left\{\left(-\frac{125}{3}+\frac{25}{2}+100\right)-\left(\frac{64}{3}+8-80\right)=\left(70 \frac{5}{6}\right)-\left(-50 \frac{2}{3}\right)\right\} \end{array}$ Subtites 5 and -4 (or their limits from part(a)) into an "integrated function" and subtracts, either way round. <br> See above working to decide to award $3^{\text {rd }}$ M1 mark here: See above working to decide to award $4^{\text {th }}$ M1 mark here: <br> So area of $R$ is $=121.5$ | M1 <br> A1ft <br> A1 <br> dM1 <br> M1 <br> M1 <br> A1 oe cao |

(b) $\quad 1^{\text {st }}$ M1 for an attempt to integrate meaning that $x^{n} \rightarrow x^{n+1}$ for at least one of the terms.

Note that $20 \rightarrow 20 x$ is sufficient for M1.
$1^{\text {st }} \mathrm{A} 1$ at least two out of three terms correctly ft . Note this accuracy mark is ft in Way 2.
$2^{\text {nd }} \mathrm{A} 1$ for correct integration only and no follow through. Ignore the use of a ' $+c$ '.
Allow $2^{\text {nd }} \mathrm{A} 1$ also for $-\frac{x^{3}}{3}+\frac{2 x^{2}}{2}+24 x-\left(\frac{x^{2}}{2}+4 x\right)$. Note that $\frac{2 x^{2}}{2}-\frac{x^{2}}{2}$ or $24 x-4 x$ only counts as one integrated term for the $1^{\text {st }} \mathrm{A} 1$ mark. Do not allow any extra terms for the $2^{\text {nd }} \mathrm{A} 1$ mark. $2^{\text {nd }}$ M1: Note that this method mark is dependent upon the award of the first M1 mark in part (b).
Substitutes 5 and -4 (and not 4 if the candidate has stated $x=-4$ in part (a).) (or the limits the candidate has found from part(a)) into an "integrated function" and subtracts, either way round. Allow one slip!
$3^{\text {rd }}$ M1: Uses the integral of $(x+4)$ with correct ft limits of their $x_{1}$ and their $x_{2}$ (usually found in part (a)) $\left\{\right.$ where $\left(x_{1}, y_{1}\right)=(-4,0)$ and $\left(x_{2}, y_{2}\right)=(5,9)$. \} This mark is usually found in the first line of the candidate's working in part (b).
$4^{\text {th }}$ M1: Uses "curve" - "line" function with correct ft (usually found in part (a)) limits. Subtraction must be correct way round. This mark is usually found in the first line of the candidate's working in part (b).
Allow $\int_{-4}^{5}\left(-x^{2}+2 x+24\right)-x+4\{\mathrm{~d} x\}$ for this method mark.
$3^{\text {rd }} \mathrm{A} 1: 121.5$ oe cao.
Note: SPECIAL CASE for this alternative method
Area of $R=\int_{-4}^{5}\left(x^{2}-x-20\right) \mathrm{d} x=\left[\frac{x^{3}}{3}-\frac{x^{2}}{2}-20 x\right]_{-4}^{5}=\left(\frac{125}{3}-\frac{25}{2}-100\right)-\left(-\frac{64}{3}-8+80\right)$
The working so far would score SPEICAL CASE M1A1A1M1M1M0A0.
The candidate may then go on to state that $=\left(-70 \frac{5}{6}\right)-\left(50 \frac{2}{3}\right)=-\frac{243}{2}$
If the candidate then multiplies their answer by -1 then they would gain the $4^{\text {th }} \mathrm{M} 1$ and 121.5 would gain the final A1 mark.

## edexcel

| Question Number | Scheme ${ }^{\text {arks }}$ |
| :---: | :---: |
| Aliter <br> 9. (a) <br> Way 2 | Curve: $y=-x^{2}+2 x+24$, Line: $y=x+4$   <br> \{Curve $=$ Line $\} \Rightarrow y=-(y-4)^{2}+2(y-4)+24$ Eliminating $x$ correctly. B1 <br> $y^{2}-9 y\{=0\} \Rightarrow y(y-9)\{=0\} \Rightarrow y=\ldots .$. Attempt to solve a resulting <br> quadratic to give $y=$ their <br> values. M1 <br> So, $y=0,9$ Both $y=0$ and $y=9$. A1 <br> So corresponding $y$-values are $x=-4$ and $x=5$. See notes below. B1ft <br>   [4] |
|  | $2^{\text {nd }}$ B1ft: For correctly substituting their values of $y$ in equation of line or parabola to give both correct ft $x$-values. |
| 9. (b) | Alternative Methods for obtaining the M1 mark for use of limits: <br> There are two alternative methods can candidates can apply for finding " 162 ". <br> Alternative 1: $\begin{aligned} & \int_{-4}^{0}\left(-x^{2}+2 x+24\right) \mathrm{d} x+\int_{0}^{5}\left(-x^{2}+2 x+24\right) \mathrm{d} x \\ = & {\left[-\frac{x^{3}}{3}+\frac{2 x^{2}}{2}+24 x\right]_{-4}^{0}+\left[-\frac{x^{3}}{3}+\frac{2 x^{2}}{2}+24 x\right]_{0}^{5} } \\ = & (0)-\left(\frac{64}{3}+16-96\right)+\left(-\frac{125}{3}+25+120\right)-(0) \\ = & \left(103 \frac{1}{3}\right)-\left(-58 \frac{2}{3}\right)=162 \end{aligned}$ <br> Alternative 2: $\begin{aligned} & \int_{-4}^{6}\left(-x^{2}+2 x+24\right) \mathrm{d} x-\int_{5}^{6}\left(-x^{2}+2 x+24\right) \mathrm{d} x \\ = & {\left[-\frac{x^{3}}{3}+\frac{2 x^{2}}{2}+24 x\right]_{-4}^{6}-\left[-\frac{x^{3}}{3}+\frac{2 x^{2}}{2}+24 x\right]_{5}^{6} } \\ = & \left\{\left(-\frac{216}{3}+36+144\right)-\left(\frac{64}{3}+16-96\right)\right\}-\left\{\left(-\frac{216}{3}+36+144\right)-\left(-\frac{125}{3}+25+120\right)\right\} \\ = & \left\{(108)-\left(-58 \frac{2}{3}\right)\right\}-\left\{(108)-\left(103 \frac{1}{3}\right)\right\} \\ = & \left(166 \frac{2}{3}\right)-\left(4 \frac{2}{3}\right)=162 \end{aligned}$ |

## Appendix

## List of Abbreviations

- dM1 denotes a method mark which is dependent upon the award of the previous method mark.
- ft or $\sqrt{ }$ denotes "follow through"
- cao denotes "correct answer only"
- aef denotes "any equivalent form"
- cso denotes "correct solution only"
- AG or * denotes "answer given" (in the question paper.)
- awrt denotes "anything that rounds to"
- aliter denotes "alternative methods"


## Extra Solutions

If a candidate makes more than one attempt at any question:

- If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
- If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

| Question <br> Number | Scheme | Marks |
| :---: | :---: | :---: |
| Aliter <br> 4. (c) <br> Way 2 | $(x+2)^{2}+(y-1)^{2}=16$, centre $\left(x_{1}, y_{1}\right)=(-2,1)$ and radius $r=4$. $d_{1}=\sqrt{4^{2}-2^{2}}=\sqrt{12}$ <br> Applying $\sqrt{\text { their } r^{2}-\mid \text { their }\left.x_{1}\right\|^{2}}$ <br> Hence, $y=1 \pm \sqrt{12}$ <br> Applies $y=$ their $y_{1} \pm$ their $d$ <br> So, $y=1 \pm 2 \sqrt{3}$ <br> $1 \pm 2 \sqrt{3}$ | M1 <br> A1 aef M1 <br> A1 cao <br> cso <br> [4] |

Special Case: Award Final SC: M1A1 M1A0 if candidate achieves any one of either $y=1+2 \sqrt{3}$ or $y=1-2 \sqrt{3}$ or $y=1+\sqrt{12}$ or $y=1-\sqrt{12}$.

| Aliter |
| :---: | :--- | ---: | :--- |
| 8. (a) |$|$| $2 x^{2}\left(\frac{L-12 x}{4}\right)=81$ | $2 x^{2}\left(\frac{L-12 x}{4}\right)=81$ |
| :--- | ---: |
| Way 2 | $\Rightarrow x^{2}(L-12 x)=162 \Rightarrow L=12 x+\frac{162}{x^{2}}$ |

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